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A descent into the maelstrom

'I became possessed with the keenest curiosity about the whirl itself. I positively felt a wish to explore its depths, even at the sacrifice I was going to make; and my principal grief was that I should never be able to tell my old companions on shore about the mysteries I should see.'

Edgar Allan Poe, 'A descent into the maelstrom' (1840)

The Kerr black hole

All the stars rotate. For this reason they are not exactly spherical, but slightly flattened at the poles. The gravitational collapse of a real star is not therefore exactly described by the spherically symmetric Schwarzschild solution. In reality, the geometry of the surrounding space-time will become much more complicated because of the production of *gravitational waves.*

Why do these gravitational waves[1] disturb the geometry? The reason is simple: all moving matter (for example a rotating star) has a gravitational field which varies with time. Consequently, the curvature it causes in space-time changes at each instant to reflect the new configuration of matter. These readjustments propagate at the velocity of light as 'wrinkles' in the curvature, travelling across the background geometry.

Collapsing stars with the least spherical shapes emit the most gravitational waves. If the star collapses into a black hole and an event horizon is formed then the situation simplifies itself in an instant. At the moment of its formation the horizon may still have

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[1] See Chapter 18.

an irregular shape and be subjected to violent vibrations; however, a fraction of a second later the gravitational waves smooth out all irregularities (Figure 34). The horizon stops vibrating and assumes a unique smooth shape: a spheroid flattened at the poles by centrifugal forces.

This is why the gravitational field of a rotating star that collapses into a black hole reaches a final state of equilibrium which depends on two parameters only: mass and *angular momentum*. The latter is related to the rotational motion of the star and is similar to the spin of elementary particles (see page 73).

There is an exact solution of Einstein's equations which depends only on these two parameters. It was discovered in 1962 by the New Zealand physicist Roy Kerr and describes the gravitational field of a rotating black hole. The astronomical implications of this theoretical discovery were considerable, comparable to the discovery of a new elementary particle. Science has always been like this, theory and experiment feeding each other.

Whereas the Schwarzschild geometry describes a gravitational field caused by a spherical mass, static or not, Kerr's geometry describes a final equilibrium solution which can only be applied once the event horizon has formed and all the distortions have been 'swept away' by gravitational waves, and not during the actual collapse of the rotating star.

The maximal black hole

Most stars are in differential rotation. They are composed of layers of gas of varying densities, which do not rotate at the same velocity. In the Solar System, the atmospheres of the gaseous planets such as Jupiter and Saturn show the effects of differential rotational by exhibiting elongated bands parallel to the equator. The Kerr black hole is rotating with perfect *rigidity*: all the points on the horizon move with the same angular velocity.

On the other hand, stars cannot rotate with just any velocity. Even neutron stars, which are like giant spinning tops, cannot perform more than a thousand rotations per second: above this limit they would disintegrate under centrifugal forces. There is a critical angular momentum beyond which the event horizon would

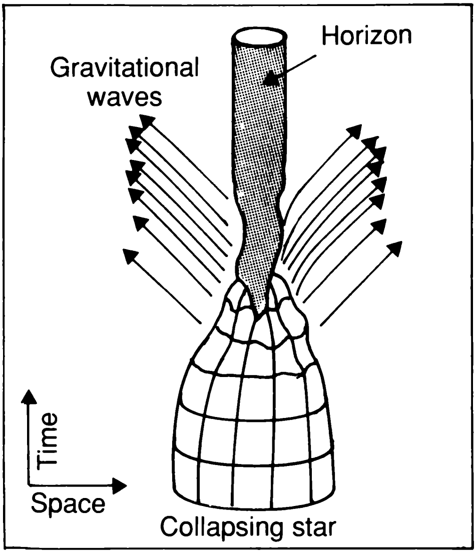

Figure 34. The formation of a non-spherical black hole.

The deformations of a collapsing star are dissipated as gravitational waves, and an axisymmetric black hole rapidly forms.

'break up' leaving the naked central singularity. This limit corresponds to the horizon having a rotational velocity equal to that of light. For such a black hole, called a 'maximal' black hole, the gravitational field at the event horizon would be zero. In Newtonian language this means that on the event horizon the force of centrifugal repulsion exactly cancels the force of gravitational attraction.

It is possible that most of the black holes formed by the gravitational collapse of a massive star have an angular momentum which is very close to this critical limit. In fact, a number of rotating stars, although far from being black holes, already have a very high angular momentum (the Sun's is 20% of the critical limit). If the angular momentum is conserved during the collapse,[2]

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[2] The conservation of the angular momentum explains the very high rotational velocities of neutron stars, see Chapter 7.

it is likely that stellar black holes approach this limiting state. Thus black holes of 3 $M_\odot$, believed to be the 'engines' of binary X-ray sources (see Part 4), must rotate at almost 5000 revolutions per second.

But a black hole is not a spinning top revolving in a fixed exterior space. We cannot fix a lamp on the horizon and count the number of passes per second. As it rotates, a Kerr black hole *drags the entire fabric of space-time along with it.*[3] Theoretically it is only at an infinite distance that space-time ceases to 'rotate' and that it is possible to attribute an angular velocity to the horizon. Closer to the black hole, space-time is irresistibly sucked into a whirlpool shape. After the capture of light, this is the second fundamental characteristic of a black hole: *it is a cosmic maelstrom.*

The cosmic maelstrom

> *'But little time will be left to me to ponder upon my destiny! The circles rapidly grow small – we are plunging madly within the grasp of the whirlpool – and amid a roaring and bellowing and thundering of ocean and of tempest, the ship is quivering – oh! God! – and . . . is going down!'*
>
> Edgar Allan Poe, 'Manuscript found in a bottle'

There is a profound analogy between a rotating black hole and the familiar phenomenon of a vortex, whether it be the swirling water which goes down a plug hole when a bath is emptied, or the giant whirlpools produced by sea currents, such as the legendary maelstrom off the coast of Norway (described by Edgar Allen Poe in his *Tales of the Grotesque and Arabesque*), or even the Corrievreckan in the Scottish Hebrides mentioned by Jules Verne in his book *The Green Ray.*[4]

In a whirlpool, water moves in a spiral which can be decomposed

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[3] General Relativity states that it is also true for all massive rotating bodies, but the dragging of the geometry, called the *Lense–Thirring effect*, is minimal unless the body has collapsed into a black hole.

[4] Also it should not be forgotten that at the end of *Twenty Thousand Leagues under the Sea*, Jules Verne caused the submarine *Nautilus* to disappear into one of these marine abysses.

into a circular motion and a suction towards the centre. The circular motion has a purely tangential velocity proportional to the inverse square of the distance from the centre of the whirlpool; the suction has a purely radial velocity which is much smaller than the tangential velocity and varies as the inverse of the distance from the centre.

Now imagine that a motor boat ventures into a whirlpool (Figure 35). We assume that the boat has a maximum velocity of 20 km/h in calm water. Far from the whirlpool the captain can obviously navigate where he wants; the motor has no problem in overcoming the motion of the water. He can thus remain in a fixed position without needing to anchor. He can approach or move away from the whirlpool, and travel against the current.

If the captain decides to navigate towards the whirlpool, there will come a point at a certain distance from the centre where the circular velocity of the current equals the maximum velocity of the boat, 20 km/h. Inside this critical distance, the boat is unable to maintain a fixed position, even with its engine running at full speed. It is forced to travel in the same direction as the rotation of the whirlpool. In other words, the boat which was originally free to travel in any direction is now limited to a region bounded by the inside of an angle closed by the straight lines joining the boat position and the tangents to the 'navigation circle' in front of the boat. However, although dragged along by the circular current, the boat can still escape from the whirlpool by orienting itself on a suitable trajectory and spiralling outwards.

If the boat travels too close to the centre of the whirlpool there will come a fatal moment when the radial velocity of the current equals 20 km/h (the circular velocity is already much greater). The navigation circles then plunge directly into the mouth of the whirlpool; as Edgar Allan Poe wrote 'if a ship comes within its attraction, it is inevitably absorbed and carried down to the bottom, and there, beat to pieces'.

The analogy with the Kerr geometry surrounding a rotating black hole is clear. The centre of the whirlpool is the black hole. The water surface pulled down by the whirlpool is the space-time curved by gravitation and drifting in the sense of the vortex. The boat could be a spaceship or any material particle, whose

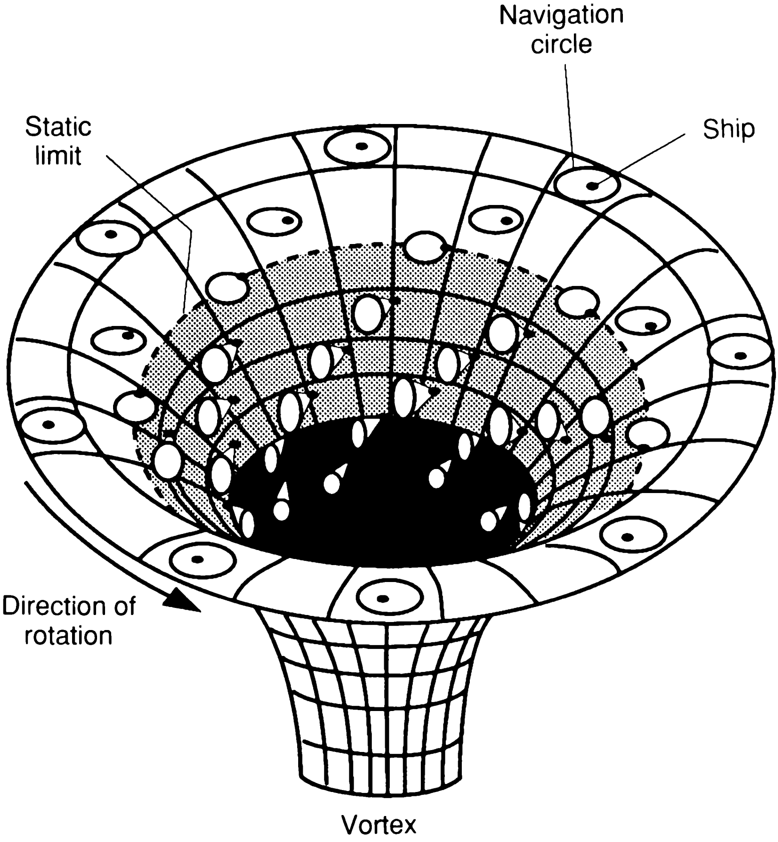

Figure 35. The black hole maelstrom.

The gravitational well caused by a rotating black hole resembles a whirlpool. A spaceship travelling in the vicinity of a black hole is sucked towards the centre of the vortex like a boat. In the region outside the static limit (clear), it can navigate where it wants. In the zone (grey) between the static limit and the event horizon it is forced to rotate in the same sense as the black hole; its ability to navigate freely is decreased as it is sucked inwards, but it can still escape by travelling in an outwards spiral. The dark zone represents the region inside the event horizon; any ship which ventured there would be unable to escape even if it was travelling at the velocity of light.

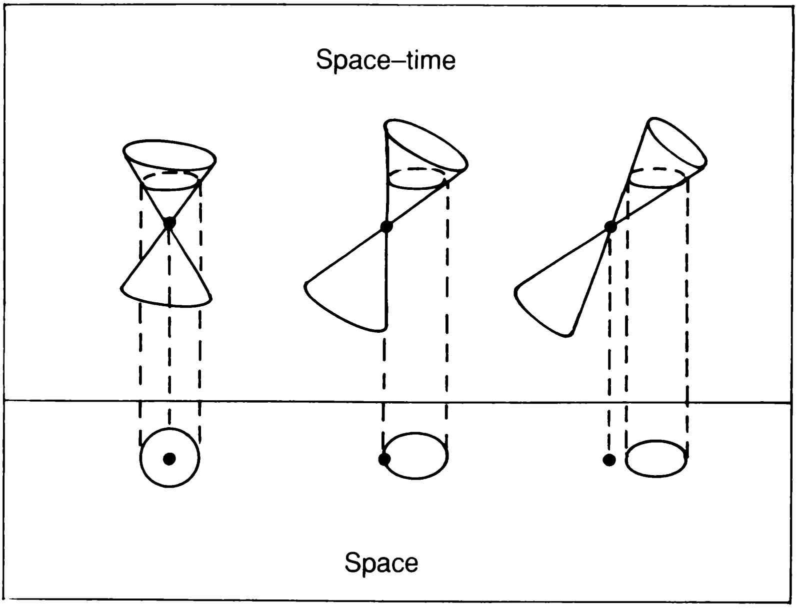

Figure 36. Navigation circles.

If we cut a light cone at fixed time (a horizontal plane in this figure) the resulting spatial section is a 'navigation circle' (or more exactly an ellipse) which determines the limits of the permitted trajectories. If the cone tips over sufficiently in the gravitational field, the navigation circle detaches itself from the point of emission. The navigational directions are confined within the angle formed by the tangents of the circle and it is impossible to return.

maximum allowed velocity is that of light, 300 000 km/s. As Figure 36 shows, the navigation circle at a given point is a spatial projection of a light cone which marks out the allowed trajectories.

The light cones are not only deviated towards the interior of the gravitational field, but are also dragged in the rotational sense of the black hole. This 'spiral' is inexorable inside what is called the *static limit*. In this region, the circles of light – the projections of the cones are detached from their emission point and shifted forwards. Consequently the spaceship is unable to remain static with respect to a distant fixed reference frame (for example the stars), even if it is travelling at the velocity of light.

Closer still to the centre of the black hole is a second critical surface where the light cones are tipped so far inwards that nothing can escape from it. We recognise the *event horizon*, the true boundary of the Kerr black hole.

The event horizon is situated inside the static limit, but these two characteristic surfaces of the Kerr black hole touch at the poles (Figure 37). They have quite distinct roles. Time appears to 'freeze' at the static limit and radiation acquires an infinite redshift, but it is only at the event horizon that matter is completely imprisoned.[5]

The region of space-time between the two surfaces is called the ergosphere. The name was invented by John Wheeler from the Greek word for 'work', because it is theoretically possible to use some of its unusual properties to extract the rotational energy of the black hole. I will return to this astonishing speculation in Chapter 13.

The ring singularity

The internal structure of a rotating black hole is much more complex than that of a static black hole. The first important difference is the central singularity, where the curvature becomes infinite. In a rotating black hole it is no longer a point but a flat *ring* in the equatorial plane. This ring is no longer a space-time knot towards which all matter must converge. It is now possible to travel inside a rotating black hole by avoiding the ring singularity, either by travelling above its plane or by passing through it! These new possibilities for exploring black holes will be discussed in Chapter 12.

There is another difference: there is a second event horizon inside the actual boundary of the black hole. This spherical surface surrounds the ring and 'protects' the region between the internal and external event horizon from the singularity.[6] As the angular

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[5] In the Schwarzschild black hole we recall that the single horizon has both properties.

[6] In the sense that a signal emitted by the singularity cannot escape from the internal horizon.

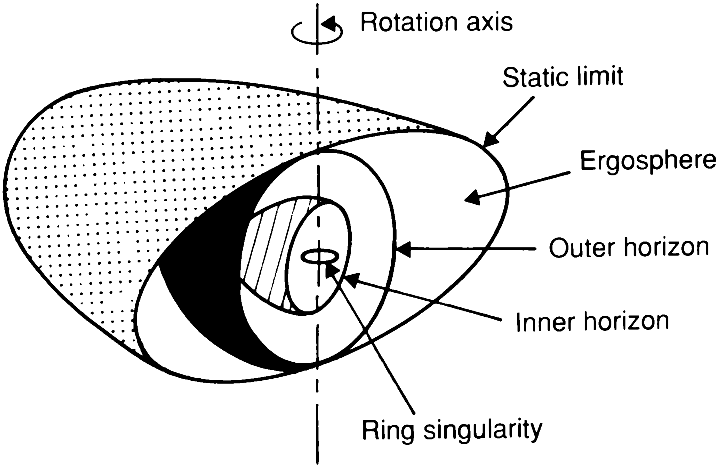

Figure 37. Cross-section of a rotating black hole. This spatial representation illustrates the complex internal structure: multiple horizons and a ring singularity.

momentum of the black hole increases, the two horizons tend to merge, the internal horizon expands and the external one contracts. At the limit, for a maximal black hole rotating at the critical speed, the two horizons break up leaving a naked gravitational singularity.

The electric black hole

Stars which collapse into black holes generally possess a magnetic field. In addition, black holes swallow electrically charged particles from the interstellar medium, such as electrons and protons. It is therefore reasonable to expect black holes to have electromagnetic properties.

H. Reissner in 1916, and independently G. Nordstrøm in 1918, discovered an exact solution to Einstein's equations for the gravitational field caused by an electrically charged mass. This solution is a generalised version of Schwarzschild's solution, with one other parameter: the electric charge. It describes space-time outside the event horizon of an *electrically charged black hole.*

If the electromagnetic properties of a black hole reduce to a single electric charge, the electromagnetic structure of the parent star (field lines, existence of magnetic poles and so on) must have simplified considerably. Here again, gravitational waves have carried away most of the electromagnetic attributes of the star, leaving behind only a global electric charge, not localised on the horizon, analogous to the electric charge of an elementary particle. This charge does not alter the shape of the black hole, which remains perfectly spherical in the absence of rotation.

There is a limit to the amount of electric charge a black hole may have. Above a critical limit the event horizon would be destroyed by the colossal force of electrostatic repulsion. The maximum electric force is proportional to the mass of the black hole, and for a 10 $M_\odot$ black hole it is 10^{40} times the charge on an electron. Nevertheless, a black hole is just as likely to be positively charged as negatively charged.

The interior structure of a highly charged black hole has features common to that of a static neutral hole or a rotating hole; as in the former, the singularity is a point, but as in the latter, it is screened by an inner event horizon. The area of the inner horizon increases and that of the outer horizon decreases as the electric charge grows. At the maximum permitted charge, the two horizons would merge together and disappear, revealing the singularity to distant astronomers.

Despite these subtleties, this discussion of highly charged black holes is fairly academic, because 'natural' black holes are probably neutral. This is for the same reason that most ordinary matter is neutral: the remarkable weakness of gravitation compared with electromagnetic interactions. A macroscopic body (that is, one containing an enormous number of elementary particles) contains almost exactly equal numbers of positive and negative charges (carried by electrons and protons). The electrostatic forces cause these charges to associate and neutralise each other. Let us now imagine that a black hole has formed with a large positive charge, near to the maximum value allowed. In a realistic astrophysical environment, the black hole will not sit in a complete vacuum but in the interstellar medium, which is full of protons and electrons. The black hole's gravitational field will attract both electrons and protons; however, its electric charge will attract only charges of the opposite sign, electrons, and will repel the protons. The electrostatic forces are a billion billion times stronger than the gravitational forces. Therefore in a very short time the black hole will have captured all the available electrons and will have almost completely neutralised itself. The electric charge of a 'natural' black hole cannot be any greater than one billion billionth of its maximum charge. This is so small that the astrophysical effects of the black hole's electric charge can be ignored.

A black hole has no hair

Are there as many types of black holes in the Universe as there are stars? In other words, apart from mass, angular momentum and electric charge, what parameters can black holes have?

For a physicist a star or a sugar cube are fantastically complicated objects, in the sense that a *complete* description of them, including their atomic and nuclear structure, requires billions of parameters. But a physicist studying the exterior of a black hole does not have the same problems. A black hole is by contrast an incredibly simple object: if we know its mass, angular momentum and electric charge, then we know *everything* there is to know about it.

A black hole retains practically nothing of the complexity of the matter which formed it. It does not remember its shape or composition; it keeps only the mass, angular momentum and electric charge (Figure 38). This disarming simplicity is perhaps the most basic characteristic of a black hole. John Wheeler, who invented most of the terminology concerning black holes, remarked in the 1960s that 'black holes have no hair'.

What began as conjecture has recently received a mathematically rigorous proof, a result of efforts over 15 years by half a dozen theoreticians, including Brandon Carter of the Observatoire de Meudon and the Australian Gary Bunting. Confirming Wheeler's statement, their work showed that only three parameters were needed to describe the geometry of space-time around a black hole *in equilibrium*. For the theoretician this implies a considerable simplification: there are only four types of black hole, depending on which parameter is the most important.[7] Let us list them: the spherical and static Schwarzschild black hole characterised by its mass; the Reissner and Nordstrøm black hole, which is also spherical and static but has an electric charge; the Kerr black hole, which is a rotating neutral mass; and finally the most general equilibrium black hole, rotating and charged, which was calculated in 1965 and given the name Kerr–Newman. This last solution represents the *unique and natural final state of a gravitational collapse within an horizon.* As we have explained, the electric charge plays a negligible role, so the most 'realistic' black hole is correctly described by the Kerr solution.

Once again gravitational waves sweep away all the complex structure of the matter as the black hole forms. They 'shave the hair' from the black hole leaving only its mass, angular momentum and electric charge. These physical parameters are characteristic of the two long-range interactions present at the formation of a black hole: gravitation (for the mass and angular momentum) and electromagnetism (for the electric charge). The short range nuclear interactions which structure atomic nuclei play no role in the formation of black holes.

The black hole parameters are perfectly measurable, albeit by means of thought experiments. A black hole could be weighed by placing a satellite in orbit around it and measuring its orbital period. Its angular momentum could similarly be measured by comparing the deviations of light rays sent towards various parts of its horizon.

For a general Kerr–Newman black hole of given mass, the electric charge and the angular momentum both have upper limits. They are constrained by a relation ensuring the existence of the event horizon. If this constraint is violated – for example by the gravitational collapse of a massive star – the black hole would become a naked singularity, capable of influencing the Universe at large distances. Physicists have good reason to believe that such a situation is prevented by the laws of Nature.[8]

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[7] The mass which is responsible for the gravitational field is of course taken into account.

[8] This important subject will be dealt with in the next chapter.

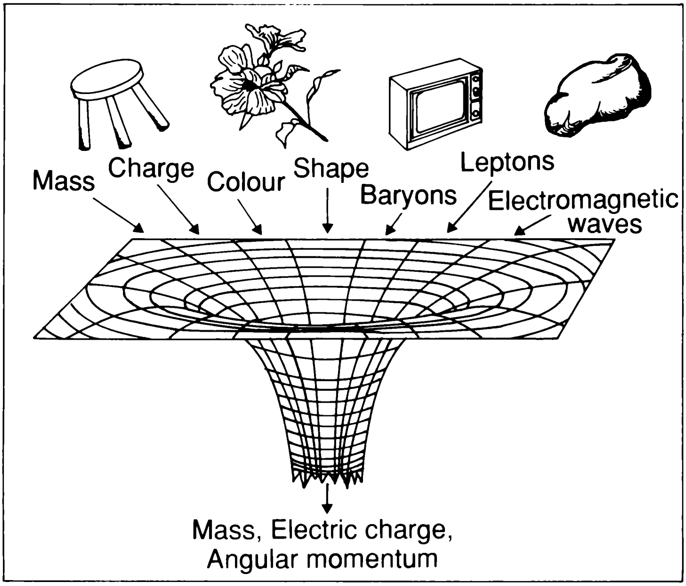

Figure 38. The black hole remembers only the mass, angular momentum and electric charge of the matter which falls into it. (After Ruffini and Wheeler.)

Since it is only affected by three parameters, a black hole is of the same order of simplicity as an elementary particle. However, if we examine the condition of the existence of an event horizon, nothing could be more unlike a black hole than an elementary particle, even though the latter assembles mass, angular momentum and charge in a very small volume. Taking the case of an electron, we know from experiment its mass, angular momentum (spin) and electric charge. For its given mass, the electric charge and angular momentum of an electron exceed the black hole limits by a factor of 10^{88}. This staggering number, which is even greater than the total number of elementary particles in the observable Universe, is a measure of the difference between an electron and a Kerr–Newman black hole![9]

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[9] Which is not to say that an electron is a naked singularity!

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Map games

'The map is not the territory.'

Alfred Korzybski

Black and white

The human mind has a natural preference for symmetry. Since antiquity, physicists have been trying to analyse the mechanisms of Nature in terms of elementary symmetry. The surprising thing is that this method has often been successful. An excellent example of this was the theoretical prediction of 'anti-particles', followed shortly after by their experimental discovery. Symmetry is more important now than ever in the most recent developments in fundamental physics.

The black hole has a symmetrical opposite, the *white hole*, a sort of gravitational outflow from a region hidden behind a horizon. Early interpretations of the white hole led to the popular idea that man could travel instantaneously from one part of the Universe to another by entering a black hole and reappearing from a white hole, having travelled through the 'throats' connecting them. Such ideas certainly increased the fascination of black holes for the general public, but reduced their credibility amongst scientists unfamiliar with General Relativity.

What is the real status of white holes? We have to re-examine the delicate problem of the relationship between the real world and its mathematical description, or between a map and the actual territory. One of the most common symmetries in the laws of physics is time reversal. In Galileo and Newton's mechanics, Fresnel's optics, Maxwell's electromagnetism and Einstein's relativity, all the equations are symmetric with respect to time.